

Multi-Objective Optimization of Grinding Processes with Two Approaches: Optimal Pareto Set with Genetic Algorithm and Multi-attribute Utility Theory

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Abstract

Optimization of grinding process is a multi-objective optimization problem. However, it traditionally has been solved as a single objective problem. In this paper, general, useful and practical multidisciplinary optimization methods are discussed for optimal design of surface grinding processes. The methods allow designer to explicitly consider and control multiple design objectives as an integrated part of the optimization process, and easily choose and set up preferences for the objectives in order to increase productivity and quality of the workpiece surface.

The methods discussed in this paper are Pareto efficient set and multi-attribute utility theory based optimization approaches. The algorithm for finding Pareto set is proposed

and an example is presented. A new formulation for multi-objective optimization of grinding process is developed. The results of the formulation represent the tradeoffs the designers are willing to make between work piece surface roughness, tool life, grinding ratio, and material removal rate. Four examples are used to illustrate the application of the formulation for multi-objective optimization of the surface grinding processes. An example for defining the preferences in the multi-objective optimization process is also presented.

1. Introduction

More and more brittle materials such as ceramics, glasses, sapphires, etc are now used to produce different parts in many industries such as aerospace industry, automotive industry, etc. In order to obtain a

high accuracy, these parts are necessary to be machined, then finishing processes like grinding, lapping, polishing, etc must be applied. Grinding is one of the most important processes for finishing brittle materials. The random distribution, random orientation, and random position and geometry of abrasive grains on the wheel surface create the difficulties of creating mathematical models.

Researches have been carried out to investigate the grinding mechanism. Werner (1983) has derived a linear relationship between the normal and tangential force by making assumptions about the wheel-workpiece contact, grit, normal and tangential force equation, hardness of the workpiece and the ploughing pressure. The stages from friction to ploughing and cutting have been described by utilizing a slip-line field which satisfies all the existing boundary conditions (Lortz, 1979). The slip-line field has been developed by starting from the velocity relation at the level of penetration cutting edge and characterizing the frictional condition at the interface between the cutting edge and the work material. Liao *et al.* (1989) utilized factorial experimental design to derive an empirical model for creep feed. The drawback of

these works is that the grinding process was considered as a single objective problem.

Optimization of the grinding process includes the determination of suitable abrasive wheel and the machining conditions. Being therefore a multi-objective optimization problem (MOP). The solution to this problem requires a good knowledge of the relationship between quantitative parameters of the grinding process and of the wheel characteristics, the grinding parameters and the dynamic behavior of the grinding machine. As discussed by Sathyanarayanan (1992), the conventional approach presented in the literature for optimization of grinding suffers from few limitations. First, a large amount of experimental data has to be generated in order to model the system response accurately in the traditional way. Second, due to the conventional serial computing process, the derived model can not adjust itself to simultaneous changes of the parameters' values. Third, the problem was considered as a single objective problem.

The goal of this paper is to develop general, useful and practical multidisciplinary optimization tools for robust design of products, systems and processes. The tools allow designer to explicitly consider and

control, as an integrated part of the optimization process, the multiple design objectives. Easily choose and set up preferences for the objectives in order to increase productivity and quality of the workpiece surface.

The paper addresses several key questions in developing multi-objective methods for the design of grinding processes.

- “How should different objectives be incorporated into one model?”
- “How should multi-objective optimization of grinding problem be formulated as a multidisciplinary design optimization problem?”
- “How should the formulated models be solved?”

The rest of the paper is organized as follows. First, a brief description of the surface grinding process and the experiment design used in supporting this study are provided. Then, a literature review of multi-objective optimization in finding Pareto set with Genetic Algorithm, and multi-attribute utility theory approach are presented. The research approach developed for solving the multi-objective optimization problem is discussed in Section 3. The algorithm for finding the Pareto set is presented and

illustrated with an example for grinding Basalt (II) with diamond wheel. Then the models based on multi-attribute utility theory for Basalt (I), Basalt (II), Ceramic (I), and Ceramic (II) are developed and solved. Finally, the conclusions based on the results obtained from the mentioned examples are drawn in Section 4.

1.1. Brief Description of Surface Grinding Process

The optimization problems we are trying to solve are concerned with the surface grinding. Surface grinding is performed on either a horizontal spindle or vertical a spindle grinding machine. The work piece is secured, unusual for ceramics, on a magnetic chuck, which is attached to the worktable. The main grinding parameters are shown in Figure 1 and listed below. Geometric description some of these parameters is shown in Figure 1 as well.

V_s – wheel wear, mm^3

V_w – total stock removal, mm^3

v_s – wheel speed, m/sec

v_w – feed rate, m/min

a – depth of cut, μm

G – grinding ratio (V_w/V_s)

Q_w – material removal rate, mm^3/min

h_{eq} – equivalent chip thickness, μm

T – life of grinding wheel (tool life), min

Ra – roughness, μm

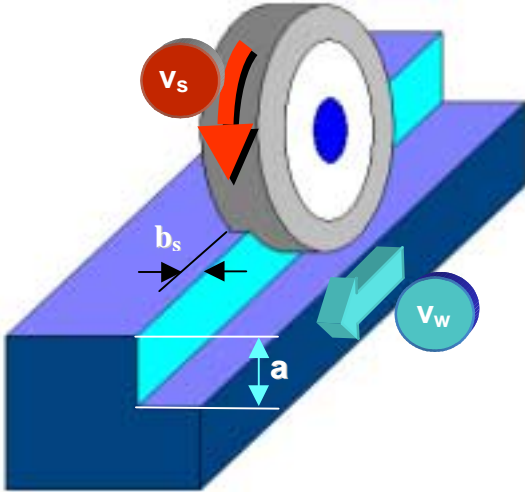


Figure 1. Illustration of the Surface Grinding Process Showing Various Process Variables

1.2. Experimental Procedure

A surface grinder type Rubin 026 VA II (ELB Schliff) is equipped with instrumentation for measuring grinding forces, vibration, velocities, diamond wheel wear, and energy consumed during wet grinding process. Details of the wheel specifications and the test condition are given in the Tables 1, 2, and 3.

Table 1. Wheel and bond characteristics

Wheel bond	Metal Bond	Resin Bond
Wheel speed	25 m/sec	25 m/sec
Workpiece speed	16 m/min	6 m/min
Crossfeed	3 mm	5 mm

Infeed	0.01 to 0.1	0.005 to 0.05
Coolant	water based synthetic emulsion 2%,	water based synthetic emulsion 2%,
Flow rate	5 l/min	5 l/min
Workpiece area	193 cm^2	193 cm^2
h_{eq} range	0.05 – 1 μm	0.05 – 0.5 μm

Table 2. Properties of the Materials used

Properties	Ceramic	Basalt
Density	3.90 g/cm^3	2.75 g/cm^3
Thermal conductivity	39 W/cm K	5 W/cm K
Bending strength	9.1 N/mm^2	7.5-8 N/mm^2
Max utilization temp.	1800 $^{\circ}\text{C}$	400 $^{\circ}\text{C}$

Table 3. Wheel Specifications

Wheel Specifications		
Characteristics	Metal Bond	Resin Bond
Type	1A1	1A1
Dimensions	300-6-3 mm	250-10-3 mm
Abrasives	Medium friable synthetic diamond	Friable Ni-coated 56% synthetic diamond
Grit size	D126	D 107
Concentration	100	100
Hardness	HB 128	R

Throughout the rest of the paper, the following parameters have been used. For grinding Basalt (I) metal bond has been used and the following characteristics were applied: wheel 1A1 300-6-3 D 126 C100 HB 128, $v_s = 30$ m/sec, $v_w = 16$ m/min, $b = 2$ mm. Basalt (II) means that the resin bond has been used and wheel 1A1 250-10-3

D107 C100 R, $v_s = 25\text{m/sec}$, $v_w = 4\text{ m/min}$, $b = 6\text{ mm}$. To process Ceramic (I) metal bond wheel has been used, wheel 1A1 250 –10-3 D 126 M100 $v_s = 25\text{ m/sec}$, $v_w = 16\text{m/min}$, $b = 3\text{ mm}$. To grind Ceramic (II) resin bond has been used and wheel 1A1 300-15-2 D76 R75, $v_s = 25\text{ m/sec}$, $v_w = 6\text{ m/min}$, $b = 5\text{ mm}$.

2. Literature Review

In this section, literature that related to two methods in solving multi-objective optimization problems is viewed. These two methods are Pareto set generation and multi-attribute utility theory. Recently several methods for solving multi-objective optimization problems have been developed. For example, Horn *et al.* (1994) modified a genetic algorithm (GA) by introducing the concept of Pareto's domination in the selection operator. In order to improve their results, they applied a niching pressure to spread population out along the Pareto optimal surface, by alerting tournament selection in the two ways. They added Pareto dominant tournaments, and when they obtained a tie the winner was determined by implementing a sharing. Consequently, they called this algorithm a niched Pareto GA. These authors compared their approach with vector evaluated GA (VEGA) which has been developed by Schaffer (1985). Only extreme points on the

Pareto front were found with VEGA, whereas the niched Pareto GA gave better results.

Viennet *et al.* (1996) has developed the multi-objective optimization algorithm that uses GA properties and a non-dominated solution selection algorithm to generate a set of Pareto efficient points. This technique permits one to determine the Pareto set with its contour line. In this method each objective is weighted and summed to form a scalar objective that is minimized or maximized. Another technique in generating Pareto points is to transform all objectives except one into constraints. With this method it is very difficult to take into the account variability of raw materials and working conditions to get good results. These techniques are neither accurate nor general. The advantage of this algorithm is that any function can be simultaneously optimized. And the result consists of an optimal oneness zone in which the decision-maker is able to choose the working conditions. This method is based on diploid genetic algorithm and on selection procedure using the pressure of Pareto. Compared with the algorithm of Horn *et al.* (1994), this pressure is not applied in the selection stage of this algorithm. The way they applied their domination criterion

would seem less efficient, and their algorithm needs more generations to define optimal Pareto set. Multicriteria optimization algorithm uses a GA only to find intermediate population. No Pareto-efficient points are eliminated from these populations with selection procedure.

Another technique to solve MOP is Multi-attribute Utility Theory. Utility theory was originally devised by von Neumann and Morgenstern (1947) for economic application and later was developed for multiple objective decision making purpose by Keeney and Raiffa (1976). Utility theory provides a formal approach in constructing a design evaluation function that represents the designer's preferences and willingness to make tradeoff between conflicting design objectives.

Similar to the application of the utility theory to decision-based design described by Gold and Krishnamurty, (1997); Olson and Moshkovich, (1995). The utility theory based multidisciplinary design optimization technique should include the following four steps:

- (1) Determination of design attributes, design constraints, and decision variables

- (2) Representation of single attribute utility function in terms of decision variables and evaluation of single attribute utility functions and preferences based on tradeoffs
- (3) Aggregation of single attribute utility functions into an overall multiple attribute utility function
- (4) Optimization of the multiple attribute utility function for selection of optimal design

A methodology for determining and evaluating single and multiple attribute utility functions in engineering design is described in Thurston (1991) and Thurston *et al.* (1991). Thurston (1991) concluded that utility theory based approach measures more accurately designer's preferences and allows for a nonlinear relation between preference and attribute level, and non-constant tradeoffs.

3. Research approach

The research in this paper consists of three tasks. The first task is to incorporate the different objectives in solving multi-objective grinding optimization problem. The next task is to formulate the multi-objective grinding process problems. The final task is to develop a general method for solving the formulated multi-objective

grinding process problem. To accomplish those tasks two approaches were considered: Pareto efficient set with Genetic Algorithm approach and multi-objective utility theory approach.

3.1. Pareto Set with Genetic Algorithm

Approach

The optimization of grinding process includes both objectives that should be minimized and objectives that should be maximized. This makes the optimization of the grinding process a very complex problem. The objectives that should be maximized include G-ratio (G), Tool life (T), and Material Removal Rate (Q_w). The objective that should be minimized is Roughness (Ra). Decision variable is the Equivalent Chip Thickness (h_{eq}) that in turn depends on wheel wear, (V_s) mm³, total stock removal, (V_w) mm³, and depth of cut (a), mm. According to Marinescu (1984), h_{eq} is defined as following:

$$h_{eq} = \frac{av_w}{60v_s}$$

Applying simple regression analysis to the data generated from the experiment, the objectives Ra , G , T , and Q_w can be expressed as functions of h_{eq} for Basalt (II) and Ceramic (II) as follows:

Basalt (II)	Ceramic(II)
$Ra = 0.4369(h_{eq}) + 6.9158$	$Ra = 0.8841 \ln(h_{eq}) + 5.7158$
$G = -99.52 \ln(h_{eq}) + 299.06$	$G = 438.17e^{-13.719(h_{eq})}$
$T = 7317.4e^{-0.3091(h_{eq})}$	$T = 2.7298(h_{eq})^{-2.0468}$
$Q_w = 572.59e^{0.138(h_{eq})}$	$Q_w = 17372(h_{eq}) + 98.98$

Characteristically, a MOP has no unique solution that can simultaneously optimize all objectives. In multi-objective decision making situation we can find Pareto optimal solution. A Pareto optimal solution, is defined as following:

A solution x to a multiple- objective problem is a Pareto optimal if no other feasible solution is at least as good as x with respect to every objective and strictly better than x with respect to at least one objective (Winston 1997).

Any point in a Pareto optimal set can become an “optimal solution” depending on decision-maker’s preferences. There is no universal standard to judge merits of the solution. Designers may choose on the basis of tradeoffs among design objectives by determining interrelationship. A Pareto optimal set carries information on those tradeoffs. Thus, an ideal way to solve a MOP is to obtain an evenly distributed subset of a Pareto optimal set, to explore this subset, and to select the preferred solution.

GAs are based on the biological evolution mechanism and Darwin's survival-of-the-fittest theory, are intelligent search methods for solving complex optimization problems. They are problem independent algorithms and can process information generated at previous stages of a search process. A search process comprises of natural selection, quick exploration, and information collection in design space. In contrast to more classical optimization methods, a GA requires no gradient information and produces multiple optima rather than a single local optimum. These characteristics make GA a powerful tool for solving multi-objective optimization problems.

To apply multicriteria optimization algorithm for the grinding process some modification should be done. The multicriteria optimization algorithm does not consider the functions with partial maximization ($\max \{f_1(x), f_2(x), \dots, f_3(x)\}$) and partial minimization ($\min \{f_1(x), f_2(x), \dots, f_3(x)\}$) problems. To overcome this limitation, the following normalized equations are used to convert the problem into a minimization one.

$$\min \bar{f}_i = \begin{cases} \frac{f_i}{(1+f_i)}, & \text{where } f_i \text{ is to be minimized} \\ \frac{1}{(1+f_i)}, & \text{where } f_i \text{ is to be maximized} \end{cases}$$

$$\min \bar{f}_i = \begin{cases} \frac{5x+3}{1+(5x+3)}, & \text{where } f_1 \text{ is to be minimized} \\ \frac{1}{1+(3x+6)}, & \text{where } f_2 \text{ is to be maximized} \end{cases}$$

Here it is assumed that the $f_i \geq 0$.

To illustrate the concept, let's consider an example. We have two objectives: maximize $f_1=3x+6$ and minimize $f_2=5x+3$. Applying the normalized equations, the two objectives are converted into a minimization objective as in the following:

To minimize $\frac{1}{1+(3x+6)}$ is the same as to

maximize $f_1=3x+6$ and to minimize $\frac{5x+3}{1+(5x+3)}$ is

same as to minimize $f_2=5x+3$.

If any constraint is used the problem should be converted into the unconstrained optimization problem by applying Lagrange penalty function. The Pareto points generation algorithm is presented below:

Pareto Points Generation Algorithm:

- Step 1. Each function is calculated for the minimum values of the other functions and the maximum value will be denoted as F_i . For example, minimum values of material removal rate and tool life

will be computed, then G ratio will be calculated for those minimum values.

Step 2. For each function, one zone Ω_i is defined by $f_i(\mathbf{x}) \leq F_i$. The solutions for every individual function construct populations P_i .

Step 3. Eliminate Pareto-inefficient points by applying multicriteria optimization. Populations are combined to form one P population. Then the solution \mathbf{x} for the first function is taken as a reference and compared with all other individuals $\bar{\mathbf{x}}$ in P, Pareto inefficient points are eliminated by using the function h_i defined by :

$$h_i = 0, \text{ if } f_i(\mathbf{x}) > f_i(\bar{\mathbf{x}}),$$

$$h_i = 1, \text{ if } f_i(\mathbf{x}) = f_i(\bar{\mathbf{x}}),$$

$$h_i = 2, \text{ if } f_i(\mathbf{x}) < f_i(\bar{\mathbf{x}}).$$

If $\sum_i h_i > 1$, then the reference is

better than the individual ($\bar{\mathbf{x}}$) so the latter will be eliminated. If it is not eliminated point ($\bar{\mathbf{x}}$) is used as reference, and so on. By applying this procedure the Pareto set could be obtained.

Multi-objective optimization eliminates points that are not Pareto-efficient. In the first two steps, the population will be created and in the third step, individuals of those populations, which are dominated, will be eliminated. Figure 2 summarizes the multicriteria optimization algorithm.

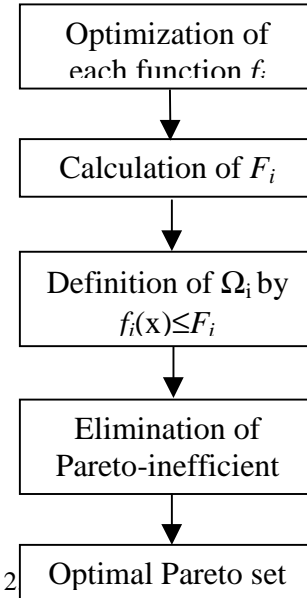


Figure 2. Optimal Pareto set i criteria optimization algorithm.

Next, the Pareto points generation algorithm is illustrated for surface grinding of Basalt (II).

Illustrative Example 1

In this example the results are computed for Basalt (II).

Step 1. All functions are normalized in the following way: The roughness of the workpiece should be minimized and the

normalization takes the following form:

$$f_1 = \frac{0.4369h_{eq} + 6.9158}{(1 + 0.4369h_{eq} + 6.9158)}$$

The other objectives should be maximized, so they take the following forms:

$$f_2 = \frac{1}{(1 + -99.52Ln(h_{eq}) + 299.06)}$$

$$f_3 = \frac{1}{(1 + 7317.4e^{-30.798h_{eq}})}$$

$$f_4 = \frac{1}{(1 + 572.59e^{0.138h_{eq}})}$$

Where f_2 is the function of G- ratio, f_3 is the function of the tool life and f_4 the function of material removal rate. To create intermediate population all functions are minimized with GA. The mortality rate of 0.8 has been used with population size of 1000. The results are presented in the Table 4.

Table 4. Results of function minimization

Functions	Solutions	
	h_{eqi}	$f_i(h_{eqi})$
$f_1(h_{eq1})$	1.001602289	0.880288266
$f_2(h_{eq2})$	1.001835818	0.003334695
$f_3(h_{eq3})$	1.020606754	0.000178079
$f_4(h_{eq4})$	20	0.000110524

Step 2. Each function is calculated for the minimum of the other three functions and the maximum values are denoted F_i . Results are presented in the Table 5.

Table 5. Functions value for different decision variable values

	$f_i(h_{eq1})$	$f_i(h_{eq2})$	$f_i(h_{eq3})$	$f_i(h_{eq4})$	F_i
Ra		0.88028973	0.88040714	0.939953644	0.939953644
G	0.003334437		0.003355366	0.519554975	0.519554975
T	0.00017703	0.000177049		0.059157469	0.059157469
Qw	0.001518684	0.001518635	0.001514713		0.001518684

Step 3. Let's consider h_{eq1} as reference point and compare with h_{eq2} , then $R_a(1.001602289) = 0.880288266 < R_a(1.001835818) = 0.88028972$, $h_1=2$
 $G(1.001602289) = 0.003334437 < G(1.001835818) = 0.003334695$, $h_2=2$
 $T(1.001602289) = 0.000177036 < T(1.001835818) = 0.000177049$, $h_3=2$
 $Q_w(1.001602289) = 0.001518684 > Q_w(1.001835818) = 0.001518635$, $h_4=2$

$h_1 + h_2 + h_3 + h_4 > 1$, so the reference h_{eq1} is better than the individual h_{eq2} hence h_{eq1} is included to the Pareto set.

Pareto set points lie on the interval [1.00, 20]. Since every objective function is either monotonically decreasing or increasing any point taken from that interval according to

the Pareto set definition will be a Pareto efficient point.

3.2. Multi-attribute Utility Theory Based

Approach

In this section, a multi-attribute utility theory based approach is developed to solve the multi-objective surface grinding optimization problem. In this approach, important design criteria are identified and weighting factors are assigned to each criterion to represent relative tradeoffs. This approach applies utility theory to assemble the important attributes into a utility function using rigorously determined scaling constants that represent the acceptable tradeoffs between attributes. When compared utility theory based approach to other approaches, Thurston (1991) concluded that utility theory based approach measures more accurately designer's preferences and allows for a nonlinear relation between preference and attribute level on one hand, and non-constant tradeoffs on the other hand.

Utility is determined as a function of performance level of attributes that a design alternative exhibits. In developing single utility functions for attributes, the preferences for the performance level of each attribute in a design are quantified by

means of lottery questions to identify certainty equivalents. These certainty equivalents are different from expected values for lotteries and reflect uncertainties and risks. Although certainty equivalents are generally unique to each individual, it is possible to characterize decision makers into risk averse or risk prone categories subject to the certainty equivalents chosen for a given lottery. The categorization of decision makers in this manner essentially implies the confidence that the decision makers have in a lottery for achieving the best possible outcome for the attribute levels under analysis. Figure 3 shows graphically the risk averse, prone, and neutral attitudes for a monotonically decreasing single attribute utility function.

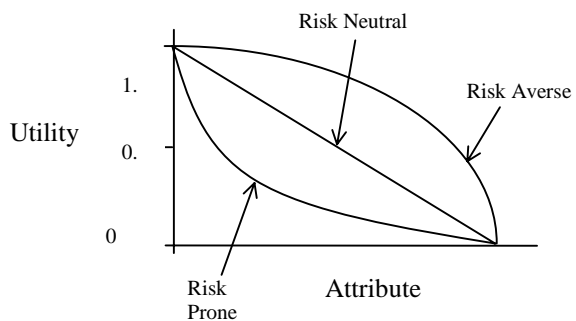


Figure 3. Graphical representation of risk attitudes

A methodology for determination and evaluation of single and multiple attribute utility functions in engineering design is described in Thurston (1991) and Thurston

et al. (1991). The relevant points in these two papers are summarized below.

Given conditions of preferential, utility, and additive independence of attributes, the overall multi-attribute utility of a design alternative is calculated with:

$$U(\mathbf{Y}) = \sum_{i=1}^n a_i U_i(y_i)$$

where:

$U(\mathbf{Y})$ = the overall utility of an alternative characterized by the attribute vector

$$\mathbf{Y} = (y_1, y_2, \dots, y_n)$$

y_i = the performance level of attribute i , $i = 1, \dots, n$

$U_i(y_i)$ = the single attribute utility function for attribute i , $i = 1, \dots, n$

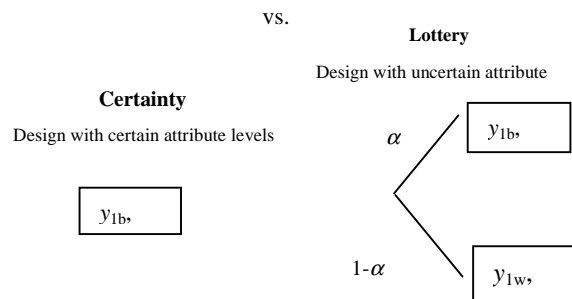
\dots, n

a_i = the single attribute

scaling constant

A detailed instruction on testing the conditions of preferential, utility, and additive independence of attributes is given in Keeney and Raiffa (1976). The single

attribute scaling constants, a_i , represent the tradeoff between attributes the designer is willing to make. These scaling constants are determined by a "lottery" type question. The value of a_i is equal to the multi-attribute utility where attribute y_i is at its best level, y_{ib} , and all other attributes are at their worst levels, y_{jw} (for all $j \neq i$). The designer is asked to compare a design with attribute levels that are certain to a design with



uncertain attribute levels, as shown in Figure 4.

On the left side, the alternative with no uncertainty in the two attribute levels is

Figure 4. Lottery questions to assess scaling constant a_1

compared with the "lottery" shown on the right side which has attribute levels dependent on probability, α . The value of α at which the designer is indifferent between the lottery and the certain attribute levels is obtained by iterating between extreme values of α . When evaluated at the best and

worst values of all attributes, \mathbf{Y}_b and \mathbf{Y}_w , the multi-attribute utility function is unity and zero, respectively. The value of a_i is then found from

$$\begin{aligned} U(y_{1w}, \dots, y_{ib}, \dots, y_{nw}) &= \alpha U(\mathbf{Y}_b) + (1 - \alpha) U(\mathbf{Y}_w) \\ &= \alpha U(1) + (1 - \alpha) U(0) \\ &= \alpha \end{aligned}$$

where $a_i = \alpha$, since $U(y_{1w}, \dots, y_{ib}, \dots, y_{nw}) = a_i$.

In solving the grinding optimization process problems, as the objective function is represented by a multiple attribute utility function, it represents all the design attributes under consideration and reflects the designers' willingness to make tradeoff between the attributes. For example, if only two attributes, roughness and material removal rate, are included in the objective function, then the optimal solution of the problem reflects the design preference over the tradeoff between the reliability and material handling cost (see Figure 5).

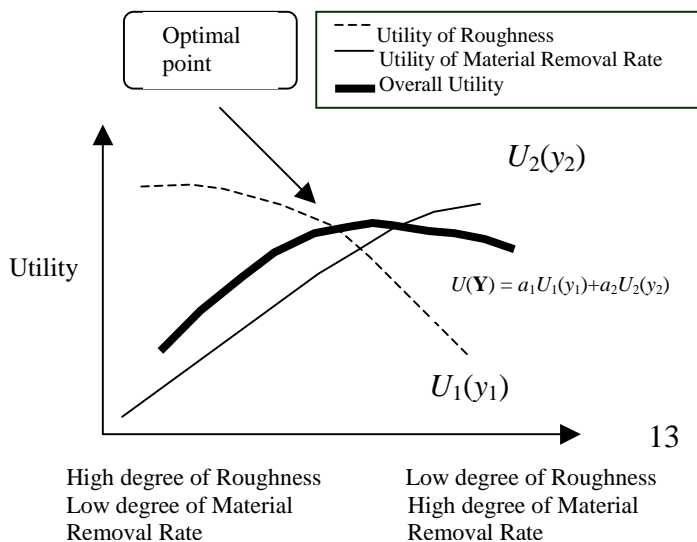
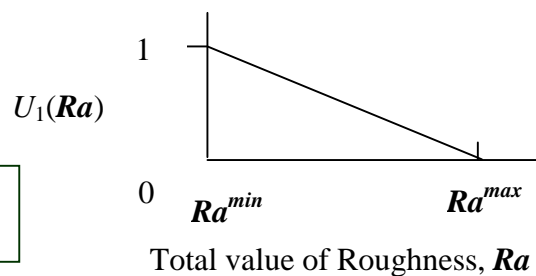


Figure 5. The overall utility when reliability decreases with decomposition of the system

The attributes are: G-ratio (G), Tool life (T), and Material removal rate (Q_w). Decision variables are: wheel wear, (V_s) mm^3 , total stock removal, (V_w) mm^3 , and depth of cut (a), mm. The single attribute utility functions of attributes take a linear form that corresponds to a neutral risk attitude. Note that the single attribute utility can also take a non-linear form depending on the decision-maker's attitude toward the risk.

Let Ra^{max} and Ra^{min} be the maximum and minimum values of the roughness, respectively. The linear relation between single utility function of the roughness $U_1(Ra)$ and the level of roughness is shown in Figure 6.



$$U_3(T) = \frac{T^{min} - T}{T^{min} - T^{max}}$$

$$U_4(Q_w) = \frac{Q_w^{min} - Q_w}{Q_w^{min} - Q_w^{max}}$$

Figure 6. Single utility function for Roughness, Ra

The single utility function of the dissimilarity measure $U_1(Ra)$ can be computed as following:

$$U_1(Ra) = \frac{Ra^{max} - Ra}{Ra^{max} - Ra^{min}}$$

Let G^{max} and G^{min} be the maximum value and minimum value of the G-ratio, respectively. The linear relation between single utility function of the G-ratio $U_2(G)$ and the value of the G-ratio is shown in Figure 7.

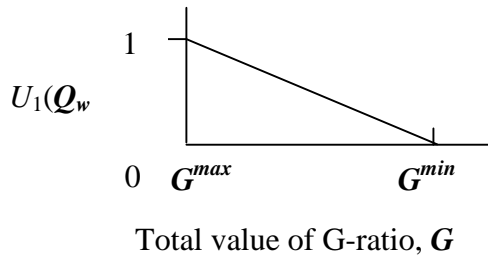


Figure 7. Single utility function for G-ratio, G

The single utility function of the G-ratio $U_2(G)$ can be computed as following:

$$U_2(G) = \frac{G^{min} - G}{G^{min} - G^{max}}$$

In a same way we can find single utility functions for Material Removal Rate (Q_w) and Tool life (T).

Given the definition of the multi-attribute utility function of the overall objective function, the optimization of grinding processes problem can be formulated as follows:

Maximize

$$U(h_{eq}) = a_1 \left[\frac{Ra^{max} - Ra}{Ra^{max} - Ra^{min}} \right] + a_2 \left[\frac{G^{min} - G}{G^{min} - G^{max}} \right] + a_3 \left[\frac{T^{min} - T}{T^{min} - T^{max}} \right] + a_4 \left[\frac{Q_w^{min} - Q_w}{Q_w^{min} - Q_w^{max}} \right]$$

Illustrative Example 2

For this example we have used material Basalt (I). The following equations were obtained from Marinescu *et al.* (1984):

$$Ra = 2.761 \left(\frac{av_w}{v_s} \right)^{0.283}$$

$$G = 382.79 \left(\frac{av_w}{v_s} \right)^{-0.8704}$$

$$T = 89.927 \left(\frac{av_w}{v_s} \right)^{-1.7085}$$

$$Q_w = 13.059 \left(\frac{av_w}{v_s} \right)^{0.84}$$

For different values of the single attribute scaling constants a_i optimal solutions were

obtained using Genetic Algorithm software. The summary of results is presented in the table 6.

Table 6. Overall Utility Function values for Basalt (I) with different values of a_i

U	h_{eq}	a_1	a_2	a_3	a_4
1	0.045	0.3333	0.3333	0.3333	0
0.78271	0.045187	0.25	0.25	0.25	0.25
0.523953	0.0045	0.16667	0.16667	0.16667	0.5
0.552429	0.89993	0.15	0.15	0.15	0.55
0.602139	0.899925	0.13333	0.13333	0.13333	0.6
0.751314	0.899978	0.08333	0.08333	0.08333	0.75
1	0.9	0	0	0	1

The same experiment was performed for Basalt (II), Ceramic (I) and Ceramic (II). The results are presented in the Tables 7, 8, 9 respectively.

The following equations for Basalt(II), Ceramic(I), and Ceramic (II) were obtained from Marinescu *et al.* (1984):

Basalt (II) Ceramic (I) Ceramic(II)

$$\begin{aligned}
 R_a &= 0.4369h_{eq} + 6.9158 & R_a &= 8.6978h_{eq}^2 - & R_a &= 0.8841\ln(h_{eq}) + \\
 G &= -99.52\ln(h_{eq}) + & & 3.3626h_{eq} + 7.23717 & & 5.7158 \\
 & 299.06 & G &= 1440.9h_{eq}^{-0.5573} & G &= 438.17e^{-13.719h_{eq}} \\
 T &= 7317.4e^{-0.3091h_{eq}} & T &= 164.86h_{eq}^{-1.4156} & T &= 2.7298h_{eq}^{-2.0468} \\
 Q_w &= 572.59e^{0.138h_{eq}} & Q_w &= & Q_w &= 17372h_{eq} + \\
 & & & 45154.4h_{eq}^{0.8745} & & 98.98
 \end{aligned}$$

Table 7. Overall Utility Function values for Basalt (II)

with different values of a_i

Computational Results					
U	h_{eq} (μm)	a_1	a_2	a_3	a_4
0.997	0.02681	0.3333	0.3333	0.3333	0
0.74803	0.02681	0.25	0.25	0.25	0.25
1					
0.49906	0.02681	0.16667	0.16667	0.16667	0.5
0.5485	0.1594	0.15	0.15	0.15	0.55
0.59825	0.1594	0.13333	0.13333	0.13333	0.6
0.74726	0.1594	0.08333	0.08333	0.08333	0.75
1					
0.9988	0.1598	0	0	0	1

Table 8. Overall Utility Function values for Ceramic (I) with different values of a_i

Computational Results					
U	h_{eq} (μm)	a_1	a_2	a_3	a_4
0.9717595	0.1109	0.3333	0.3333	0.3333	0
0.7415571	0.1084	0.25	0.25	0.25	0.25
0.499801	1.0629	0.16667	0.16667	0.16667	0.5
0.549776	1.065	0.15	0.15	0.15	0.55
0.599816	1.0658	0.13333	0.13333	0.13333	0.6
0.749596	1.0658	0.08333	0.08333	0.08333	0.75
1					
0.99923	1.0658	0	0	0	1

Table 9. Overall Utility Function values for Ceramic (II) with different values of a_i

Computational Results					
U	h_{eq} (μm)	A_1	a_2	a_3	a_4
0.9857205	0.02031185	0.3333	0.3333	0.3333	0
0.7397235	0.02031185	0.25	0.25	0.25	0.25
0.499659	0.19998444	0.16667	0.16667	0.16667	0.5

0.549967	0.19998444	0.15	0.15	0.15	0.55
0.599955	0.19998444	0.13333	0.13333	0.13333	0.6
0.749994	0.19998444	0.08333	0.08333	0.08333	0.75
0.9998637	0.19999	0	0	0	1

3.3. Definition of Preferences

In this section we try to optimize a_i coefficients for any given value h_{eq} , which will allow to find out whether the objective function is dependable on a_i coefficients. This method has the following advantages: 1) completely eliminates the need for iterative weight setting, i.e. once the designer's preferences are articulated, obtaining the corresponding optimum design is a nonnutritive process, hence improve the computational efficiency; 2) provides the means to reliable employ commercial optimization tools with minimal prior knowledge.

The optimization model is defined as follows:

$$\begin{aligned} \text{Max} \quad & U = a_1 U_1 + a_2 U_2 + a_3 U_3 + a_4 U_4 \\ \text{Subject to:} \quad & \sum_{i=1}^4 a_i = 1 \\ & 0 \leq a_i \leq 1 \end{aligned}$$

The following results were obtained for Basalt (II):

h_{eq}	a_1	a_2	a_3	a_4	U
0.0267	0.69	0.12	0.08	0.11	0.89
0.04	0.96	0.03	0	0.01	0.8882
0.055	1	0	0	0	0.7875
0.08	0.96	0.02	0	0.02	0.5917
0.1	0.04	0.02	0	0.94	0.5381

From the results we can conclude that the better the quality of surface the lesser the utility of the tool life. The best value of the utility function is obtained when the h_{eq} is the smallest, which means the smallest depth of cut a since for this experiment feed rate v_w and wheel speed v_s were fixed.

4. Conclusions

The surface grinding problems traditionally have been solved as a single objective optimization problem. As such process designs generated with such solution approach may omit other import design attributes of the process and thus result in process that performs poorly.

This paper presents two methods for solving the multi-objective surface grinding optimization problems. The first method finds Pareto efficient set and second method applies multi-attribute utility theory in solving the problems. The algorithm for finding Pareto set was developed and an example was presented. A new formulation

for multi-objective optimization of grinding process was developed and represents the tradeoffs the designers are willing to make between roughness, tool life, and material removal rate. Four examples were used to illustrate the application of the formulation for solving multi-objective grinding optimization problems. The example for definition of the preferences was also presented.

According to the computational results the following conclusions can be drawn:

- (1) The Pareto set for h_{eq} lies for Basalt (I) in $[0.01\mu\text{m}:1\mu\text{m}]$, for Basalt $[0.01\mu\text{m}:0.2\mu\text{m}]$, for Ceramic (I) in $[0.1\mu\text{m}:2\mu\text{m}]$, for Ceramic (II) in $[0.01\mu\text{m}:0.25\mu\text{m}]$. Our results show that the Pareto set lies between extreme points. Any solution, which lies in that interval, is good with respect at least to the one objective. For example if we will pick up $h_{eq}^1 = 0.065\mu\text{m}$ and $h_{eq}^2 = 0.65\mu\text{m}$ we see that h_{eq}^1 increases the values of the objective functions for tool life, G-ratio and material rate but cause simultaneous decrease in the value of roughness objective function. The h_{eq}^2 solution decreases the values of objective functions for tool life, G-

ratio and material rate while cause simultaneous increase in the value of objective function for roughness.

- (1) According to the Utility theory the preferences should be given to the material removal rate. For all kinds of materials the Utility function value is the highest when the single attribute scaling constant for the material rate $a_4=1$: $U(\text{Basalt (I)}) = 1$, $h_{eq} = 0.9\mu\text{m}$; $U(\text{Basalt (II)}) = 0.9988$, $h_{eq} = 0.159\mu\text{m}$; $U(\text{Ceramic (I)}) = 0.9992$, $h_{eq} = 1.06\mu\text{m}$; $U(\text{Ceramic (II)}) = 0.9998$, $h_{eq} = 0.1\mu\text{m}$.
- (3) For the fixed h_{eq} the result could vary. The best value of Utility function was achieved for Basalt (II) when $h_{eq} = 0.267\mu\text{m}$, and the single scaling constants for roughness, G-ratio, tool life, and material removal rate respectively are $a_1=0.69$, $a_2=0.12$, $a_3=0.08$, $a_4=0.11$, $U=0.86$. That means, in this case the preferences should be give to the roughness. So the definition of the preferences method allow as setting up preference toward objectives.

Multi-attribute utility theory is very flexible to allow adding as many objectives as necessary for determining the best design model for grinding process. The tools allow designer to explicitly consider and control, as an integrated part of the optimization process, the multiple design objectives, easily choose and set up preferences for the objectives in order to increase productivity and quality of the workpiece surface. Besides it is possible to incorporate more decision variables.

The future directions of the research will include incorporating other important design variables such as V_s – wheel wear, mm^3 , V_w – total stock removal, mm^3 , v_s – cutting speed, m/sec, v_w – feed rate, m/min, a – depth of cut and other.

References

- Cheng, F. Y., and Li, D., 1997, “Multiobjective Optimization Design with Pareto Genetic Algorithm”, *Journal of Structural Engineering*, Vol. 123, No. 9, pp. 1252 - 1261.
- Horn, J., Nafpliotis, N., and Goldberg, D. E., 1994, “Niche Pareto Genetic Algorithm for Multiobjective Optimization”, *IEEE Conference on Evolutionary Computation – Proceedings*, Vol. 1, p 82 - 87.
- Keeney, R. L., and Raiffa, H., 1976, *Decision with Multiple Objectives: Preferences and Value Tradeoffs*, Wiley and Sons, New York.
- Liao, T. W., Sathyanarayanan, G., Storer, R. H., and Wilson, G. R., 1989, “Off-line Optimization of Creep Feed Grinding of Aluminum Ceramics” *Grinding Fundamentals and Applications*, edited by S. Malkin and J. A. Kovach, ASME PED 39.
- Lortz, W., 1979, “A model of cutting mechanism in grinding”, *Wear*, Vol. 53, pp. 115-128
- Marinescu I., 1984, “Tribo-Technological Aspects of Ceramic Diamond Grinding”, Working Paper, Department of Mechanical, Industrial, and Manufacturing Engineering, University of Toledo, Toledo, Ohio.
- Schaffer, J. D., 1985, “Multiple Objective Optimization with Vector Evaluated Genetic Algorithms”, in GREFFENSTETTE, J. (Ed.) *Proceedings of International Conference on Genetic Algorithms and Their Applications*, pp. 93 –100.

Thurston, D. L., 1991, "A Formal Method for Subjective Design Evaluation with Multiple Attributes," *Research in Engineering Design*, Vol. 3, pp. 105 - 122.

Thurston, D. L., Carnahan, J. V., and Liu, T., 1991, "Optimization of Design Utility," *Proceedings of ASME Design Theory and Methodology Conference*, Miami, Florida, pp. 173 - 180.

Viennet, R. Fonteix, C., Marc, I., 1996, "Multicriteria Optimization Using A Genetic Algorithm for Determining A Pareto Set", *International Journal of Systems Science*, Vol. 27, No. 2, pp. 255 - 260.

von Neumann, J., and Morgenstern, O., 1947, *The Theory of Games and Economic Behavior*, 3rd Edition, Princeton University Press, Princeton, NJ.

Werner, G., 1983, "Increased Removal Rates and Improved Surface Integrity by Creep Feed Grinding", *AES Magazine*, May-June, pp. 4 - 10

Winston, W., L., 1997, *Operations research Applications and Algorithms*, 3rd edition, PWS-KENT Publishing Company, Boston, MA.